THE EDUCARE (SIROHI CLASSES) TEST SERIES 2018

XII PHYSICS TEST

MODERN PHYSICS

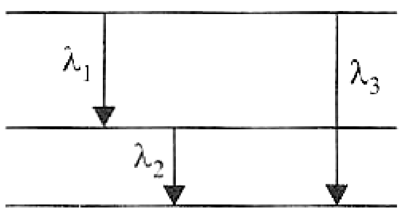
NAME-…………………………...........…   DATE-………..   MM- 25 TIME-1 HR

1) Write one equation representing nuclear fusion reaction. (1)
2) Arrange radioactive radiations in the increasing order of their ionising power. (1)
3) Name the series of the hydrogen spectrum which lies in the visible region of the electromagnetic spectrum. (1)
4) A radioactive isotope decays in the following sequence :

$$A \xrightarrow{{}^{1}_{0}n} A_1 \xrightarrow{\alpha} A_2$$

If the mass number and atomic number of A_2 are 176 and 71 respectively, find the mass number and atomic number of A and A_1. (1)

5) The ground state energy of hydrogen atom is –13.6 eV. What are the kinetic and potential energies of the electron in this state ? (2)
6) The de Broglie wavelength of a particle of kinetic energy K is λ. What would be the wavelength of the particle if its kinetic energy were $\frac{K}{4}$? (2)
7) The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of the n = 2 and n = 3 orbits ? (2)
8) Define atomic mass unit. Find its energy equivalent in MeV. (2)
9) Find the relation between the three wavelengths λ_1, λ_2 and λ_3 from the energy level diagram shown in Fig. (2)



10) The total energy of an electron in the first excited state of the hydrogen atom is about – 3.4 eV.

a) What is the kinetic energy of the electron in this state ?

b) What is the potential energy of the electron in this state ?

c) Which of the answers above would change if the choice of the zero of potential energy is changed ?

d) Calculate the wavelength of light emitted if an electron makes a transistion to the ground state. [Ground state energy = – 13.6 eV] (3)

11) Draw the graphs showing the variation of photoelectric current with anode potential of a photocell for (i) the same frequencies but different intensities $I_1 > I_2 > I_3$ of incident radiation, (ii) the same intensity but different frequencies $\nu_1 > \nu_2 > \nu_3$ of incident radiation. Explain why the saturation current is independent of the anode potential. (3)

OR

What is Photoelectric effect ? State its laws and derive these laws from Einstein's Photoelectric equation.

12) State the basic postulates of Bohr's theory of atomic spectra. Hence obtain on expression for the energy of the orbital electron of hydrogen atom. (5)

SOLUTIONS

1) Write one equation representing nuclear fusion reaction. (1)

SOL:

$$ {}_1^2H + {}_1^2H \longrightarrow {}_2^4He + 24\ MeV. $$

2) Arrange radioactive radiations in the increasing order of their ionising power. (1)

SOL: γ -rays , β -rays, α -rays

3) Name the series of the hydrogen spectrum which lies in the visible region of the electromagnetic spectrum. (1)

SOL: Balmer series lies in the visible region of the electromagnetic spectrum.

4) A radioactive isotope decays in the following sequence :

$$ A \xrightarrow{{}_0^1 n} A_1 \xrightarrow{\alpha} A_2 $$

If the mass number and atomic number of A_2 are 176 and 71 respectively, find the mass number and atomic number of A and A_1. (1)

SOL: The mass number and charge number of α -particle are 4 and 2 respectively. So, the mass number and charge number of A_l are 176 + 4 *i.e.*, 180 and 71 + 2 = 73 respectively.

Again, the mass number and charge number of neutron are 1 and 0 respectively. So, mass number and charge number of A are 180 + 1 *i.e.*, 181 and 73 + 0 *i.e.*, 73.

5) The ground state energy of hydrogen atom is –13.6 eV. What are the kinetic and potential energies of the electron in this state ? (2)

SOL:

Energy, $E = -\frac{ke^2}{2r}$

Kinetic energy, $E_k = \frac{ke^2}{2r}$

Potential energy, $E_p = -\frac{ke^2}{r}$

Clearly, $E_k = -E$ and $E_p = 2E$

Now, $E = -13.6$ eV

$\therefore$ $E_k = -(-13.6 \text{ eV}) = \mathbf{13.6\ eV}$

$E_p = 2(-13.6 \text{ eV}) = -\mathbf{27.2\ eV}$

6) The de Broglie wavelength of a particle of kinetic energy K is λ. What would be the wavelength of the particle if its kinetic energy were $\frac{K}{4}$? (2)

SOL:

$$\lambda = \frac{h}{\sqrt{2mK}}$$

$$\lambda' = \frac{h}{\sqrt{2m\frac{K}{4}}} = 2\frac{h}{\sqrt{2mK}} \quad \text{or} \quad \lambda' = 2\lambda$$

7) The radius of the innermost electron orbit of a hydrogen atom is 5.3 ×10^{-11} m. What are the radii of the n = 2 and n = 3 orbits ? (2)

SOL:

We know that $r_n \propto n^2$

$$\frac{r_2}{r_1} = \left(\frac{2}{1}\right)^2 = 4$$

or $r_2 = 4r_1 = 4 \times 5.3 \times 10^{-11} \text{ m} = 2.12 \times 10^{-10} \text{ m}$

$= \mathbf{2.12\ \text{Å}}$

Again, $\frac{r_3}{r_1} = \left(\frac{3}{1}\right)^2 = 9$

or $r_3 = 9 \times 5.3 \times 10^{-11}$ m

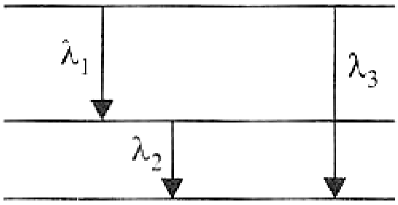
$= 4.77 \times 10^{-10} \text{ m} = \mathbf{4.77\ \text{Å}}$

8) Define atomic mass unit. Find its energy equivalent in MeV. (2)

SOL: Atomic mass unit is defined as $\frac{1}{12}$ th of the mass of one ${}^{12}_{6}C$ atom.

$$E = mc^2$$
$$= 1.66 \times 10^{-27} \times 3 \times 10^8 \times 3 \times 10^8 \text{ J}$$
$$= 1.66 \times 9 \times 10^{-11} \text{ J}$$
$$= \frac{1.66 \times 9 \times 10^{-11}}{1.6 \times 10^{-3}} \text{ MeV} = 931 \text{ MeV.}$$

9) **Find the relation between the three wavelengths λ_1, λ_2 and λ_3 from the energy level diagram shown in Fig.** **(2)**



Ans: $\frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}$

10) **The total energy of an electron in the first excited state of the hydrogen atom is about – 3.4 eV.**

a) **What is the kinetic energy of the electron in this state ?**

b) **What is the potential energy of the electron in this state ?**

c) **Which of the answers above would change if the choice of the zero of potential energy is changed ?**

d) **Calculate the wavelength of light emitted if an electron makes a transistion to the ground state. [Ground state energy = – 13.6 eV]** **(3)**

SOL:

$$\text{In Bohr's model, } mvr = \frac{nh}{2\pi} \text{ and } \frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$$

$$\text{which gives } E_k = \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}; \; r = \frac{4\pi\varepsilon_0 h^2}{Ze^2 m}n^2$$

These relations have nothing to do with the choice of the zero of potential energy. Now, choosing the zero of potential energy at infinity, we have

$$E_p = -\frac{Ze^2}{4\pi\varepsilon_0 r} \text{ which gives } E_p = -2E_k$$

$$\text{and} \qquad E = E_k + E_p = -E_k$$

(a)The quoted value of E = – 3.4 eV is based on the customary choice of zero of potential energy at infinity. Using

$E_p = -E_k$, the kinetic energy of electron in this state is + 3.4 eV.

(b) Using $E_P = -2\,E_k$, potential energy of the electron

is -2×3.4 eV = – 6.8 eV

(c) If the zero of potential energy is chosen differently, kinetic energy does not change. Its value is + 3.4 eV. This is independent of the choice of the zero of potential energy. The potential energy, and the total energy of the state, however, would alter if a different zero of the potential energy is chosen.

(*d*) $h\nu = (-3.4)\,\text{eV} - (-13.6)\,\text{eV}$
$= 10.2\ \text{eV}$
$= 10.2 \times 1.6 \times 10^{-19}\ \text{J}$

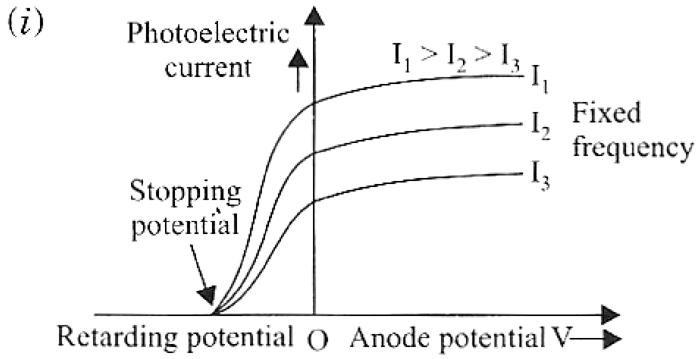
$$\frac{hc}{\lambda} = 10.2 \times 1.6 \times 10^{-19}\ \text{J}$$

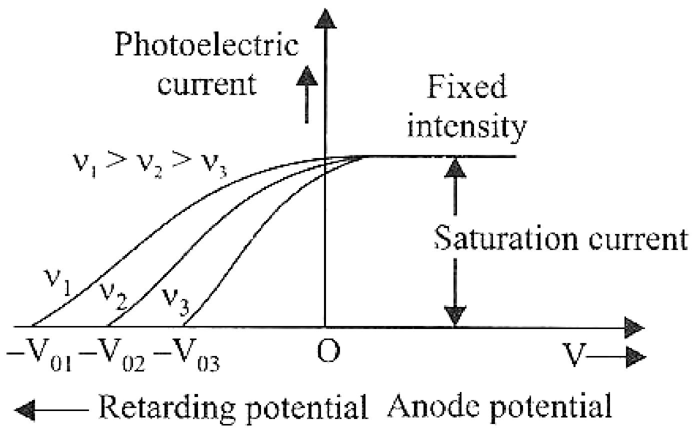
$$\lambda = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{10.2 \times 1.6 \times 10^{-19}}\ \text{m} = 1217\ \text{Å}$$

11) Draw the graphs showing the variation of photoelectric current with anode potential of a photocell for (i) the same frequencies but different intensities $I_1 > I_2 > I_3$ of incident radiation, (ii) the same intensity but different frequencies $\nu_1 > \nu_2 > \nu_3$ of incident radiation. Explain why the saturation current is independent of the anode potential. (3)

SOL:

(*i*)





The saturation current corresponds to the case when all the electrons emitted by the emitter are collected by the collector. So, any change in collector *i.e.*, anode potential will not affect saturation current.

OR

What is Photoelectric effect ? State its laws and derive these laws from Einstein's Photoelectric equation.

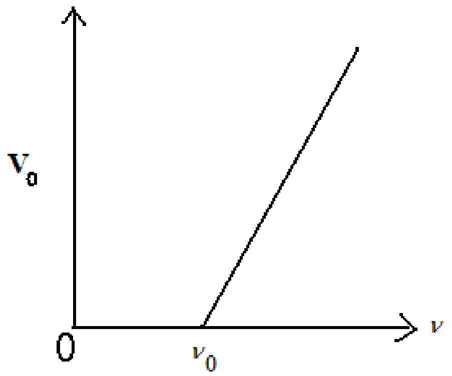
SOL: **Photo-Electric Effect:**

The photo-electric effect is the emission of electrons (called photo-electrons) when light strikes a surface. To escape from the surface, the electron must absorb enough energy from the incident radiation to overcome the attraction of positive ions in the material of the surface.

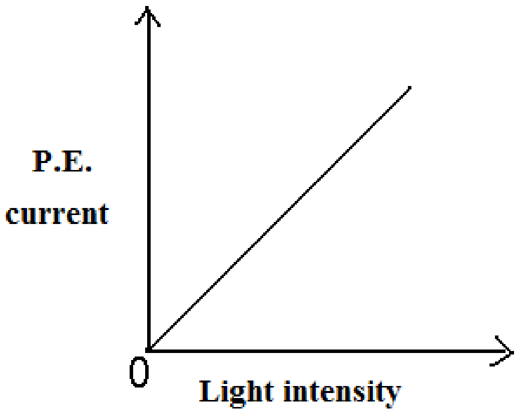
The photoelectric effect is based on the principle of conservation of energy.

Laws of photo electric emission

1) The emission of photoelectrons takes place only when the frequency of the incident radiation is above a certain critical value , characteristic of that metal .this minimum frequency is known as **threshold frequency.**



2) The emission of photo electrons starts as soon as light falls on metal surface(the time leg is less than 10^{-9} sec between the incidence of light and emission of photo electrons.)
3) The number of photo electrons emitted (photo electric current) from a metal surface depends only on the intensity of incident light and is independent of its frequency.



4) The Maximum kinetic energy with which photoelectrons are emitted from a metal surface depends only upon the frequency of the incident light and is independent of its intensity.

EINSTEIN'S PHOTOELECTRIC EQUATION

To explain photoelectric effect, Einstein postulated that the energy carried by a photon of radiation of frequency ν is h ν. According to him, the emission of a photoelectron was the result of the interaction of a single photon with an electron, in which the photon is completely absorbed by the electron.

The minimum amount of energy required to eject an electors out of the metal surface is called the work function of the metal. It is denoted by ω. Thus, when a photon of energy h ν is absorbed by an electron, an amount of energy at least equal to ω (provided $h\nu > \omega$) is used up in liberating the electron free and the difference h $\nu > \omega$ becomes available to the electron as its maximum kinetic energy. Thus,

$$\frac{1}{2} m\, v_{max}^2 = h\nu - \omega$$

or

$$h\nu = \omega + \frac{1}{2} m\, v_{max}^2 \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

Here, in is the mass of electron and ν_{max} is the maximum velocity of the photoelectrons. In fact, most of the electrons possess kinetic energy less than the maximum value as they lose a part of their kinetic energy due to collisions in escaping from the metal. Further, the work function of the metal is a characteristic of the metal and does not depend upon the nature of the incident radiation. It is sometimes also called the threshold energy of the metal. If ν_0 is the frequency which corresponds to threshold energy of the metal, then $\omega = h\nu_0$

Here, υ_o is called the threshold frequency. In the equation (1), substituting $\omega = h\nu_0$, we obtain

$$h\nu = h\nu_0 + \frac{1}{2} m\, v_{max}^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The above relation is called the Einstein's photoelectric equation.

Let us now deduce laws of photoelectric emission from the Einstein's photoelectric equation.

1) From the equation (2), the value of maximum kinetic energy of the emitted photoelectrons is given by

$$\frac{1}{2} m\, v_{max}^2 = h\nu - h\nu_0 \qquad \ldots\ldots\ldots\ldots..(3)$$

For photoelectric emission to take place, the kinetic energy of the emitted electrons must be positive. From the equation (3), it follows that the photoelectrons will possess positive kinetic energy only if h ν > h ν_0 or if $\nu > \nu_0$. It proves that for photoelectric emission to take place, the frequency of the incident radiation must be greater than the threshold frequency for the metal. If the frequency of the incident radiation is below the threshold frequency for the metal, no photoelectric emission will take place, no matter how intense the incident radiation may be or how long it falls on the metal surface

2) From the equation (3), it follows that the value of maximum kinetic energy of the emitted photoelectrons depends linearly on the frequency.- It proves that the maximum kinetic energy of the emitted photoelectrons increases, as the frequency of the incident radiation is increased. Since the Einstein's equation does not involve a factor presenting intensity of the incident radiation, it proves that the maximum kinetic energy of the emitted photoelectrons is independent of the intensity of the incident radiation.

3) According to Einstein, the photoelectric effect arises, when a single photon is absorbed by a single electron i.e. it is a one photon-one electron phenomenon. Therefore, number of photoelectrons ejected will be large, when intense radiation is incident. It is because, intensity of radiation is proportional to number of photons per unit area per unit time. Therefore, an intense radiation will contain a large number of photons and likewise, the number of photoelectrons emitted will also be large. It proves that the number of photoelectrons emitted depends on the intensity of incident radiation. Further, there is no effect of frequency of the incident radiation on the number of photoelectrons emitted. It is because, one photon is capable of ejecting only one electron, provided $\upsilon > \upsilon_0$.

4) According to Einstein, the basic process involved in photoelectric emission is absorption of a photon of light by an electron. Therefore, as the photon is absorbed, the emission of electron takes place instantaneously. It may be noted that irrespective of the intensity of the incident radiation, photoelectric emission is instantaneous.

12) State the basic postulates of Bohr's theory of atomic spectra. Hence obtain on expression for the energy of the orbital electron of hydrogen atom. (5)

SOL: <u>Postulates of Bohr's atom model</u>

1. In a hydrogen atom, the negatively charged electron revolves in a circular orbit around the heavy positively charged nucleus. The centripetal force required by the electron is provided by the attractive force exerted by the nucleus on it.

2. The electron can revolve round the nucleus only in those circular orbits in which the angular momentum of an electron is integral multiple of $h / 2\pi$, where h is Planck's constant

($= 6.62 \times 10^{-34}$ j s).

While revolving in such a orbit, the electron cannot radiate energy. Such orbits are called non-radiating or stationary orbits. Each stationary orbit is associated with a definite amount of energy. An electron revolves in a stationary orbit without radiating energy. This concept overcomes the main difficulty of Rutherford's atom model i.e. it saves electrons from falling into the nucleus.

If m and v are mass and velocity of the electron in a permitted orbit of radius r, then

$$m v r = n \frac{h}{2\pi}, \qquad \ldots\ldots\ldots\ldots\ldots(3)$$

where n is called the principal quantum number and it has the integral values 1, 2, 3,……

The equation (3) is called Bohr's quantisation condition.

3. The energy is radiated, when an electron jumps from higher to lower energy orbit and the energy is absorbed, when it jumps from lower to higher energy orbit.

If E_i and E_f are the energies associated with the orbits of principal quantum numbers n_i and n_f respectively ($n_i > n_f$), then the frequency (ν) of the radiation emitted is given by

$$h\nu = E_i - E_f \quad \ldots\ldots(4)$$

The equation (4) is called Bohr's frequency condition.

BOHR'S THEORY OF HYDROGEN ATOM

In a hydrogen atom, an electron having charge - e revolves round the nucleus having charge + e in a circular orbit of radius r as shown in Fig.

The electrostatic force of attraction between the nucleus and the electron is given by

$$F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e \times e}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r^2} \quad \ldots\ldots(5)$$

If m and v are mass and orbital velocity of the electron, then the centripetal force required by the electron to move in circular orbit of radius r is given by

$$F_c = \frac{m v^2}{r} \quad \ldots\ldots(6)$$

The electrostatic force of attraction (F_e) between the electron and the nucleus provides the necessary centripetal force (F_e) to the electron.

Therefore, from the equations (5) and (6), we have

$$\frac{m v^2}{r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r^2}$$

$$m v^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r} \quad \ldots\ldots(7)$$

According to Bohr's quantisation condition, angular momentum of the electron,

$$m v r = n\frac{h}{2\pi}$$

$$v = \frac{n h}{2\pi m r} \quad \ldots\ldots(8)$$

Putting the value of v in the equation (7), we have

$$m\left(\frac{n h}{2\pi m r}\right)^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r}$$

$$r = 4\pi\varepsilon_0\cdot\frac{n^2 h^2}{4\pi^2 m e^2} \quad \ldots\ldots(9)$$

(Bohr's Radius)

Substituting h= 6.6×10^{-34} J .s , m = 9.1×10^{-31} kg,

$4\pi\varepsilon_0 = \dfrac{1}{9\times10^9}$; e = 1.6×10^{-19} C

$$r = n^2 \times 5.29\times10^{-11} m$$

$$r = 0.529\,\mathring{A} \simeq 0.53\,\mathring{A}$$

$$r \propto n^2$$

Since n =1, 2, 3, 4...., from the equation (9), it follows that the radii of the stationary orbits are proportional to n^2 and the radii increase in the ratio 1 : 4 : 9 :16...., from the first orbit.

In the equation (8), substituting the value of r, we have,

$$v = \frac{n\,h}{2\,\pi\,m}\left(\frac{1}{4\,\pi\,\varepsilon_0}\cdot\frac{4\,\pi^2 m\,e^2}{n^2\,h^2}\right)$$

$$v = \frac{1}{4\,\pi\,\varepsilon_0}\cdot\frac{2\,\pi\,e^2}{n\,h} \qquad \text{……………(10)}$$

$$v = \frac{1}{4\pi\varepsilon_0}.\frac{2\pi e^2}{nh}\times\frac{c}{c}$$

$$v = \left(\frac{1}{4\pi\varepsilon_0}.\frac{2\pi e^2}{ch}\right)\times\frac{c}{n} \qquad where\left(\frac{1}{4\pi\varepsilon_0}.\frac{2\pi e^2}{ch}\right) = \alpha = \textbf{ Fine structure constant } = \left(\frac{1}{137}\right)$$

$$v = \alpha\times\frac{c}{n} = \left(\frac{1}{137}\right)\times\frac{c}{n} \qquad \textbf{For Hydrogen atom n =1}$$

$$v = \left(\frac{1}{137}\right)c$$

The equation (10) gives velocity of electron in the nth orbit.

<u>Energy of electron</u>: Let E_k and E_P be respectively the kinetic and the potential energies of the electron in the nth orbit, whose radius (r) is given by the equation

$$\text{Obviously,} \qquad E_k = \frac{1}{2}\,m\,v^2$$

Using the equation (7), we have

$$E_k = \frac{1}{4\,\pi\,\varepsilon_0}\cdot\frac{e^2}{2\,r}$$

The electrostatic potential energy of the electron (having charge - e) revolving in a circular orbit of radius r round the nucleus (having charge + e) is given by

$$E_p = \frac{1}{4\,\pi\,\varepsilon_0}\cdot\frac{(+e)(-e)}{r} = -\frac{1}{4\,\pi\,\varepsilon_0}\cdot\frac{e^2}{r}$$

The total energy of electron revolving round the nucleus in the orbit of radius r is given by

$$E = E_k + E_p = \frac{1}{4\,\pi\,\varepsilon_0}\cdot\frac{e^2}{2\,r} + \left(-\frac{1}{4\,\pi\,\varepsilon_0}\cdot\frac{e^2}{r}\right)$$

$$E = -\frac{1}{4\,\pi\,\varepsilon_0}\cdot\frac{e^2}{2\,r}$$

It may be pointed out that in the above equation, r represents the radius of the nth orbit Therefore, the above equation is the expression for the energy of the electron in the nth orbit. From the equation (9), substituting the value of r in the above equation, we have

$$E_n = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{2}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{4\pi^2 m e^2}{n^2 h^2}\right)$$

$$E_n = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2\cdot\frac{2\pi^2 m e^4}{n^2 h^2}$$

……….(11)